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### FAILURE PROGNOSIS USING NONLINEAR SHORT-TIME PREDICTION AND MULTI-TIME SCALE RECURSIVE ESTIMATION

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#### ABSTRACT

In this paper we describe a new, general purpose machinery diagnostic/prognostic algorithm for tracking and predicting evolving damage using only available “macroscopic” observable quantities. The damage is viewed as occurring in a hierarchical dynamical system consisting of a directly observable, “fast” subsystem coupled with a hidden, “slow” subsystem describing damage evolution. This method provides damage diagnostics and failure prognostics requiring only the measurements from the fast subsystem and a model of the slow subsystem. Damage tracking is accomplished by a two-time-scale modeling strategy based on phase space reconstruction using the measured fast-time data. Short-time predictive models are constructed using the reconstructed phase space of the reference (undamaged) fast subsystem. Later, fast-time data for the damaged system is collected and used to estimate the short-time reference model prediction error, or a tracking function. An average value of the tracking function over a given data record is used as a tracking metric, or measure of the current damage state. Recursive, nonlinear filtering is used to estimate the actual damage state based on the tracking metric input. Estimates of remaining useful life are obtained recursively using a linear Kalman filter. This method is applied to an experimental nonlinear oscillator containing a beam with a crack which propagates to complete failure. We demonstrate the ability to track the evolving damage state of the beam using only strain time series data. We also give accurate predictions of remaining useful life, in real time, beginning well in advance of

the final complete fracture at the end of experiment.

#### INTRODUCTION

To stay competitive in today’s rapidly developing economy, industry strives to wholly utilize the useful life of their products and machinery without sacrificing environmental, personnel or consumer safety. The development of condition based maintenance and failure prediction technology is addressing this important efficiency and safety issue. The problem is complicated by the fact that most processes responsible for system failures are hidden from observer. In many cases, we do not know the appropriate damage physics or how to describe it mathematically. Even when damage physics is known, it is usually difficult, if not impossible, to get a direct measure of the damage state without removing machinery from operation.

A comprehensive review of damage identification and system health monitoring is presented by Doebling et al. (1996, 1998). A more recent review article of methods relating to health monitoring of composite structures is presented by Zou et al. (2000). In addition to these papers, in the following paragraphs we give a brief representative overview of main developments in the field. For clarity, it is instrumental to identify the two main steps in the damage identification problem – damage diagnosis and prognosis. Diagnosis encompasses damage detection, localization, and assessment. Damage prognosis is contingent on successful damage assessment, i.e., if we successfully estimate the

current damage state, then given a suitable damage evolution law, we can predict when the system will fail or damage state reach a predetermined failure value.

Most of the earlier work on damage identification focuses on diagnosis, and, in particular, damage detection. Several strategies exist for tackling this type of problem (see Qu et al. 1993, and Ma and Li 1995, for comparative studies). One data-based, or heuristic, approach is to look for changes due to the damage accumulation in time or frequency domain statistics (see, for example, Worden et al., 2000, Pai and Jin 2000, Messina et al., 2000, Rivola and White, 1998, McFadden and Smith 1985, Robert and Lawrence, 1986), or in statistics that have both time and frequency information (see, for example, Luo et al., 2000, Lin and Qu, 2000, Liu et al. 1997, Staszewski et al., 1997, Wang and McFadden, 1996, 1995, McFadden and Wang 1993). For nonlinear systems exhibiting chaotic response it is customary to use estimates of long-time chaotic invariant measures, such as a correlation dimension (e.g., Craig et al., 2000, Jiang et al., 1999, Logan and Mathew, 1996a,b). The main advantages of such methods are simplicity of implementation and that they can work very well at times. Most heuristic methods serve as purely damage detection methods, i.e. no damage state assessment is provided. Even when the severity of damage can be estimated (see Ruotolo and Surace, 2000), it is usually very hard to establish a direct one-to-one connection between the damage state and the change in the heuristic statistic or feature vector. There is no theoretical basis for predicting *a priori*, without the benefit of a good model or experiment, whether a certain feature vector will work well, or not, for a particular system.

Another model-based approach addresses some of the shortcomings of the purely statistical approach, typically at the expense of more difficult development and implementation (see, for example, Iserman 1984, Gertler 1993, Natke and Cempel 1997, and references therein). In cases where the system's analytical model is available it is usually possible to establish a functional connection between the drifting parameters and a particular feature vector (Nijmeijer and Fossen 1999). However, due to the general difficulty of developing physics-based mathematical models for many systems, such analytical models are not usually available.

The lack of analytical model is usually addressed by developing finite element or data-based models. For linear systems, for example, the use of autoregressive model parameters spectra for fault detection in ball bearings is investigated by Dron et al. (1998), or Thyagarajan et al. (1998) and Sampaio et al. (1999) use frequency response functions for damage detection. Nonlinear systems are usually modeled using neural networks (Worden, 1997, McGhee et al., 1997, Alessandri and Parisini, 1997, Li and Fan, 1997), expert systems and fuzzy logic (e.g., Mechefske, 1998, Liu et al. 1996, Li and Elbestawi, 1996) and genetic algorithms (Ruotolo and Surace, 1997, Jeon and Li, 1995). Other successful approaches are based on some type of hybrid meth-

ods. For example, extensive attention is allotted to the use of mode shapes or their curvatures for damage detection and identification (see for example, Cornwell et al., 1999, Sampaio et al., 1999, Ray and Tian, 1999, Abdel Wahab and Roeck, 1999). However, in many cases these methods are application dependent, and the main advantage of a model-based approach to correlate the changes in a feature vector with the changes in a system's physical parameters is lost.

The failure prognostics problem is still in the developmental stages. Currently available prognostic methods can be divided into methods based on deterministic (e.g., Billington et al., 1999) and probabilistic or stochastic (e.g., Li et al. 2000, Swanson et al., 2000) modeling of fault or damage propagation. The given methods are still application dependent, since they are closely tied to a particular damage detection problem.

In this paper we present a damage diagnostics/prognostic method that overcomes many of the aforementioned limitations using an abstract mathematical formulation of the damage evolution problem in state space. Here, we implicitly treat the hidden process as evolving in a hierarchical dynamical system consisting of a "fast-time" subsystem coupled to a "slow-time" subsystem with the form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\mu}(\boldsymbol{\phi}), t), \quad (1a)$$

$$\dot{\boldsymbol{\phi}} = \epsilon \mathbf{g}(\mathbf{x}, \boldsymbol{\phi}, t), \quad (1b)$$

where:  $\mathbf{x} \in \mathbb{R}^n$  is the fast dynamic variable (we assume that we can measure some functional combination of its components);  $\boldsymbol{\phi} \in \mathbb{R}^m$  is the slow dynamic variable ("hidden" damage state); the parameter vector  $\boldsymbol{\mu} \in \mathbb{R}^p$  is a function of  $\boldsymbol{\phi}$ ;  $t$  is time; and  $0 < \epsilon \ll 1$  defines the time scale separation between the fast dynamics and slow drift. If  $\epsilon$  were exactly zero, then  $\boldsymbol{\mu}(\boldsymbol{\phi}_0) = \boldsymbol{\mu}_0$  would be a constant vector of parameters in Eq. (1a); since  $\epsilon$  is very small, we might consider Eq. (1a) to be a model for a system with slowly drifting parameters.

The method is appropriate for systems where hidden processes evolve on a much slower time scale than the observable dynamics that can be measured on-line in real time. This time scale separation is common for dynamical systems with evolving damage processes. For example, crack growth in a spinning shaft can be characterized by a time scale of hours, days, weeks or even years, while a time scale of a shaft vibration signature is characterized by milliseconds or seconds.

The diagnostics/prognostic algorithm is a full state space method and is a natural extension of a previously presented experimental method for hidden variable tracking (see Chelidze et al., 1999, Cusumano, et al., 1999). It consists of three major steps. The first step consists of a nonlinear short-time prediction module that uses phase space reconstruction to develop a reference data-based model for the fast subsystem Eq. (1a) in its initial damage (healthy) state. This reference model is then

used to generate reference model prediction error statistics with all future system fast-time data. The second step uses these error statistics to obtain a measure of the damage state,  $\phi$ , using a linear recursive estimator (Kalman filter). The third, and final, step uses the output of the damage tracker to estimate the actual damage state and predict the remaining time to failure using nonlinear and linear recursive estimators, respectively. In current implementations the damage (described by Eq. 1b) is assumed to evolve according to a piecewise power-law model, the parameters of which can be estimated on line as part of the process (here, they are assumed to be known *a priori*).

After describing the algorithm in some detail in the next section, the experimental system is presented. The basic system consists of the well-known two-well magneto-mechanical oscillator used extensively by the authors and others (see, for example, Moon and Holmes 1979, and Feeny et al. 2000). In our experiment, a small notch is initially cut into the root of the beam near its clamp. This notch develops into a crack which grows during subsequent motions until the beam fractures. The data fed into the algorithm consists only of strain signals from the beam: no crack size information is used.

## DESCRIPTION OF THE ALGORITHM

In this section, we present our damage tracking and failure prediction algorithm. After presenting the theoretical basis for the method, describe the algorithms major components in turn.

### Theoretical Basis – Tracking Function

To illustrate the basic idea on the method, let us consider a dynamical system, Eqs. (1), with a fixed damage state vector  $\phi$  ( $\epsilon = 0$ ). For fixed initial conditions specified by a time coordinate  $t_0$  and a space coordinate  $\mathbf{x}_0$ , at time  $t_0 + t_p$  we have

$$\mathbf{x}(t_0 + t_p) = \mathbf{X}(\mathbf{x}_0, t_0, t_p; \boldsymbol{\mu}(\phi)). \quad (2)$$

If we fix  $t_0, t_p$  and  $\mathbf{x}_0$ , then  $\mathbf{X}$  is purely a function of  $\boldsymbol{\mu}(\phi)$  that describes a  $p$ -parameter surface in  $\mathbb{R}^n$  space.

In principle, the value of  $\mathbf{X}(\boldsymbol{\mu}(\phi))$  can be evaluated for every  $\phi$ , and given certain yet to be determined observability conditions one can construct mappings  $\mathbf{X}^{-1}(\boldsymbol{\mu}) : \mathbb{R}^n \rightarrow \mathbb{R}^p$  or  $\mathbf{X}^{-1}(\phi) : \mathbb{R}^n \rightarrow \mathbb{R}^m$  required for true tracking. Therefore, if the function  $\mathbf{X}(\boldsymbol{\mu}(\phi))$  can be determined experimentally, subsequently it can be used to determine  $\boldsymbol{\mu}(\phi)$  or  $\phi$  provided the observability conditions are satisfied.

Now, let us consider a case of  $\epsilon \neq 0$ . For  $0 < \epsilon \ll 1$ ,  $\phi$  cannot be treated as a parameter and, in this case, the complete state of the system (Eqs. 1) is the direct product  $(\mathbf{x}, \phi)$ . However, for  $t_p \ll 1/\epsilon$  we can assume that  $\phi = \phi_0 + \mathcal{O}(\epsilon)$ . Here and thereafter,  $\odot = \mathcal{O}(\epsilon)$  means  $\lim_{\epsilon \rightarrow 0} \frac{\odot}{\epsilon}$  exists, and is finite. Now,

the state of the fast subsystem can be written as follows:

$$\begin{aligned} \mathbf{x}(t_0 + t_p; \epsilon) &= \mathbf{X}(\mathbf{x}_0, \boldsymbol{\mu}(\phi_0), t_0, t_p), \\ &= \mathbf{X}(\mathbf{x}_0, t_0, t_p; \boldsymbol{\mu}(\phi_0)) + \mathcal{O}(\epsilon), \\ &= \mathbf{X}(\mathbf{x}_0, t_0, t_p; \boldsymbol{\mu}(\phi_R)) + \frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi} (\phi_0 - \phi_R) \\ &\quad + \mathcal{O}(\|\phi_0 - \phi_R\|^2) + \mathcal{O}(\epsilon), \end{aligned} \quad (3)$$

where on the third line we have Taylor expanded expression about  $\phi_R = \phi(t_R)$ , the reference (healthy) value of the damage state variable for some reference time  $t_0 = t_R$ . Then, for fixed coordinates  $\mathbf{x}_0$ , and  $\phi_R$ , and fixed  $t_p$ , we define a *tracking function* as follows:

$$\begin{aligned} \mathbf{e} &\equiv \mathbf{X}(\mathbf{x}_0, \boldsymbol{\mu}(\phi_0), t_0, t_p) - \mathbf{X}(\mathbf{x}_0, \boldsymbol{\mu}(\phi_R), t_0, t_p), \\ &= \frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi} (\phi_0 - \phi_R) + \mathcal{O}(\|\phi_0 - \phi_R\|^2) + \mathcal{O}(\epsilon), \end{aligned} \quad (4)$$

where the coefficient matrices in the Taylor expansion are evaluated with the initial conditions  $\mathbf{x}(t_0) = \mathbf{x}_0$ , and  $\phi_R$ .

Now, it is clear that the observability condition is that the matrix  $\frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi}$  must have maximal rank. That is, we should have  $n > p > m$ , in addition to  $\text{Range}(\frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}}) \in \mathbb{R}^n$  and  $\text{Range}(\frac{\partial \boldsymbol{\mu}}{\partial \phi}) \in \mathbb{R}^p$  having dimensions  $p$  and  $m$ , respectively.

Assuming that the linear operator in the first term of the Taylor expansion Eq. (4) has maximal rank, we expect the output of the tracking function to be an affine transformation of the change in the damage variable (*linear observability*):

$$\mathbf{e} = \mathbf{C}(\mathbf{x}_0, t_0, t_p) \phi_0 + \mathbf{c}(\mathbf{x}_0, t_0, t_p, \epsilon), \quad (5)$$

where,  $\mathbf{C} = \frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi}$  is an  $n \times m$  matrix, and  $\mathbf{c} = -\frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi} \phi_R + \mathcal{O}(\|\phi_0 - \phi_R\|^2) + \mathcal{O}(\epsilon)$  is an  $n \times 1$  vector; all terms are evaluated using the initial conditions  $\mathbf{x}(t_0) = \mathbf{x}_0$ , and  $\phi_R$ .

In an experimental context, however, the procedure illustrated above may be prone to errors. It is usually impossible to repeatedly start the system from the same initial condition. Furthermore, the observability conditions may depend on  $\mathbf{x}_0, t_0$  and  $t_p$ , and so we might attempt to somehow use many values of  $\mathbf{x}_0$  and/or  $t_0$  and/or  $t_p$  to deal with this repeatability problem as well as to increase the robustness of the method. Such an approach, where the behaviors of two systems are compared for many different values of starting times, initial conditions and observation times, is philosophically similar to a model-based approach to tracking.

To actually calculate the tracking function  $\mathbf{e}$ , we need to know how the fast subsystem evolves over the time interval  $t_p$  for the current value of  $\phi_0$ , as well as how the subsystem *would*

have evolved for the reference value of  $\phi_R$ , both evolving from same initial condition  $\mathbf{x}_0$ . Since the system's fast time behavior for the current damage state characterized by  $\phi_0$  is directly measurable (i.e., we can reconstruct the fast dynamics using a sensor measurement from the fast subsystem), the strategy used in this paper is to compare it to the predictions of a *reference model* describing the fast subsystem behavior for the  $\phi = \phi_R + \mathcal{O}(\epsilon)$ .

## Reference Model Construction

By design, the system considered in this paper exhibits chaotic motion in its reference state. This was done to exploit the well-known fundamental property of chaotic attractors of exploring a fairly large region of phase space, thus making recorded chaotic trajectories useful for data-based modeling. It should be emphasized, however, that this use of chaos to obtain the reconstructed reference phase space should be viewed as an experimental and theoretical convenience: in principle, the same tracking procedure described in the following sections could be applied to study arbitrary global behaviors (including multiple coexisting periodic orbits and random excitation) for a variety of systems, both linear and nonlinear.

For clarity of discussion, we have to specify what kind of data is needed for the algorithm. We assume that scalar time series measured from the fast subsystem Eq. (1a) are sampled with sampling time  $t_s$  and collected in records of size  $t_D = Mt_s$ . The total data collection time  $t_T$  does not necessarily equal  $N_T t_D$ , where  $N_T$  is the total number of data records:  $t_T \geq N_T t_D \gg t_s$  since there may be time gaps between the data records. We also assume that the rate of damage accumulation is very slow, so that, in particular,  $t_D \ll 1/\epsilon$ . Thus, the total change in the damage variable  $\phi$  is very small over any data collection time span  $t_D$  and so for any  $t_0 \in t_D$  we can write  $\phi = \phi_0 + \mathcal{O}(\epsilon) \approx \phi_0$ . Thus, given this assumption of quasistationarity over any single data collection interval, in what follows we drop the subscript zero from  $\phi$ .

Using delay reconstruction (Takens 1981, and Sauer et al. 1991) the scalar time series  $\{x(n)\}_{n=1}^M$  is used to reconstruct a phase space of appropriate dimension (say,  $d$ ) for the system. In this reconstructed phase space the scalar time series is converted to a series of vectors  $\mathbf{y}^T(n) = (x(n), x(n+\tau), \dots, x(n+(d-1)\tau)) \in \mathbb{R}^d$ , where  $\tau$  is a suitable delay parameter. Embedding parameters  $\tau$  and  $d$  are determined using the first minimum of average mutual information (Fraser and Swinney, 1986) and false nearest neighbors (Kennel et al., 1992) methods, respectively.

The reconstructed state vectors are governed by an as yet undetermined map of the form  $\mathbf{y}(n+1) = \mathbf{P}(\mathbf{y}(n); \phi)$ . The observation period  $t_p$  now corresponds to picking an integer  $k$  ( $t_p = kt_s$ ). We examine the  $k^{\text{th}}$  iterate of the map  $\mathbf{P}$ :

$$\mathbf{y}(n+k) = \mathbf{P}^k(\mathbf{y}(n); \phi). \quad (6)$$

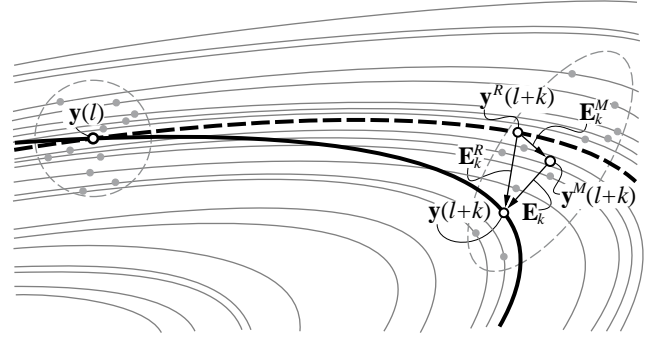


Figure 1. Schematic drawing illustrating the basic ideas about estimating the tracking function. Trajectories of the reference system in phase space are shown in thin gray lines. The solid black line represents a measured trajectory of the perturbed system, passing through  $\mathbf{y}(l)$ , and the thick dashed gray line represents where a trajectory would have gone in the reference system, if starting from the same point  $\mathbf{y}(l)$ .

Working in the global reconstructed phase space of the dynamical system, local models which show how neighborhoods about each data point are mapped forward in time can be estimated. The simplest such model is a local *linear* model relating the state at time  $n$ ,  $\mathbf{y}(n)$  to the state  $\mathbf{y}(n+k)$  at time  $n+k$ :

$$\mathbf{y}(n+k) = \mathbf{A}_n^k \mathbf{y}(n) + \mathbf{a}_n^k = \mathbf{B}_n^k \hat{\mathbf{y}}(n), \quad (7)$$

where the model parameter matrix  $\mathbf{A}_n^k$  and parameter vector  $\mathbf{a}_n^k$  are determined by regression at each point in the data set, and where

$$\hat{\mathbf{y}}(n) = [\mathbf{y}^T(n), 1]^T, \text{ and } \mathbf{B}_n^k = [\mathbf{A}_n^k | \mathbf{a}_n^k]. \quad (8)$$

## Short-Time Prediction Error Statistic

The basic ideas of tracking function estimation are shown schematically in Fig. 1. First, we collect a large reference data set and reconstruct the initial phase space (reference trajectories are shown in gray). Then, we experimentally obtain a large number of initial points  $\mathbf{y}(l)$  ( $l = 1, 2, \dots$ ), and the corresponding points after  $k$  steps,  $\mathbf{y}(l+k)$ , for the “changed” system. We look up  $N$  points in the reference set that are nearest neighbors of each point  $\mathbf{y}(l)$  and corresponding  $N$  reference data points  $k$  steps later. Based on these points we build the reference model for each point  $\mathbf{y}(l)$  as described in the previous section. In the reference system the same initial points  $\mathbf{y}(l)$  would have been mapped to points  $\mathbf{y}^R(l+k)$ . Since we have an imperfect reference model, it actually maps these same points to some other points  $\mathbf{y}^M(l+k)$ .

We introduce the following “error” vectors (for clarity, the

dependence of the  $\mathbf{E}$ 's on  $l$  is suppressed in the figure):

$$\mathbf{E}_k^R(l) := \mathbf{y}(l+k) - \mathbf{y}^R(l+k), \quad \text{the true error,} \quad (9a)$$

$$\mathbf{E}_k(l) := \mathbf{y}(l+k) - \mathbf{y}^M(l+k), \quad \text{the estimated error,} \quad (9b)$$

$$\mathbf{E}_k^M(l) := \mathbf{y}^M(l+k) - \mathbf{y}^R(l+k), \quad \text{the modeling error.} \quad (9c)$$

The tracking function (Eq. 4) for initial point  $\mathbf{y}(l)$  in the reconstructed phase space can be estimated as

$$\begin{aligned} \mathbf{e}_k(l) &= \mathbf{P}^k(\mathbf{y}(l), \phi) - \mathbf{P}^k(\mathbf{y}(l), \phi_R) = \mathbf{E}_k^R(l), \\ &= \mathbf{E}_k(l) + \mathcal{O}(\mathbf{E}_k^M(l)). \end{aligned} \quad (10)$$

We are interested in estimating the slow damage variable  $\phi$ , which requires an estimate of  $\mathbf{e} = \mathbf{E}_k^R$ . However, we can only measure the error  $\mathbf{E}_k(l)$ . Thus, what is needed is a method for estimating the true error  $\mathbf{E}_k^R$  given only  $\mathbf{E}_k(l)$ . Furthermore, the above equation uses the information at only one point in the phase space (as indicated by the index  $l$ ), whereas we wish to use all of the data available to us within one data record. Both of these issues can be addressed by an appropriate filter, which is the subject of the next subsection.

### Damage Tracking Metric

In this paper we focus on scalar damage variables. Therefore, for the actual scalar *estimated tracking function* we use  $\|\mathbf{E}_k^R(l)\|$ . However, to use the data from an entire data record, we consider instead the *tracking metric*

$$e_k = \langle \|\mathbf{E}_k^R(l)\| \rangle, \quad (11)$$

where  $\langle \odot \rangle$  represents *root mean square* (RMS) value of  $\odot$  over an entire data record of time span  $t_D$ .

We can then attempt to estimate  $\|\mathbf{E}_k^R(l)\|$  using only the measurable  $\|\mathbf{E}_k(l)\|$  time series with an appropriate filter  $\mathcal{F}$ , so that Eq. (11) becomes:

$$e_k = \langle \mathcal{F}(\|\mathbf{E}_k(l)\|) \rangle. \quad (12)$$

One way to design the filter  $\mathcal{F}$  would be to make use of an explicit analytical model for the fast subsystem Eq. (1a), which could be used to derive a model describing the dynamics of  $\|\mathbf{E}_k(l)\|$ . In our case, we have assumed that such a model is not available. However, considering that the fast sampling time  $t_s \ll t_D \ll 1/\epsilon$ , a constant model should provide an acceptable approximation for the tracking function evolution over the sampling time interval  $t_s$ . That is, over the relatively short sampling time, the best estimate of the future ‘‘error state,’’  $\|\mathbf{E}_k^R(l+1)\|$ ,

is its current value,  $\|\mathbf{E}_k^R(l)\|$ , which is, obviously, a linear relationship.

For linear systems the Kalman filter (Grewal and Angus, 1993) provides the optimal minimum mean square error estimator, that can be calculated recursively. Specifically, for  $\mathcal{F}$  we consider Kalman filter designed for random constant parameter estimation. The linear process difference equation for the scalar tracking function  $E(l) \equiv \|\mathbf{E}_k^R(l)\|$  with a scalar measurement function  $z(l) \equiv \|\mathbf{E}_k(l)\|$  is:

$$\begin{aligned} E(l+1) &= E(l) + w(l+1), \\ z(l+1) &= E(l+1) + v(l+1), \end{aligned} \quad (13)$$

where the process (model) noise  $w(l)$  and measurement noise  $v(l)$  are assumed to be white, independent random variables with Gaussian distributions  $p(w) \sim N(0, Q)$  and  $p(v(l)) \sim N(0, R(l))$ .

In this work, the constant  $Q$  corresponds to the average amplitude of fluctuations in  $\|\mathbf{E}_k^R(l)\|$  as the location in phase space changes along a given trajectory. In addition,  $R(l)$  corresponds to fluctuations in  $\|\mathbf{E}_k^R(l)\|$  due to changes in the local linear model accuracy from point to point in phase space. We take  $R(l) \propto r_l^{d_f}$ , where  $r_l$  is the distance from the point  $\mathbf{y}(l)$  to the farthest of all  $N$  nearest neighbors used for local modeling, and  $d_f$  is the estimated dimension of the data in the reference phase space, i.e., average pointwise dimension (see Ott, 1993, pg. 86). The argument behind this choice is that the accuracy of the local linear models is proportional to the generalized volume occupied by the fixed number of nearest neighbors, and that the volume by itself scales like the  $r_l$  raised into the  $d_f$  power.

The tracking function filter time update equations are:

$$\begin{aligned} \hat{E}^-(l+1) &= \hat{E}(l), \\ P^-(l+1) &= P(l) + Q. \end{aligned} \quad (14)$$

where,  $\hat{E}^-(l+1)$  is the *a priori* estimate of  $E(l+1)$  given the knowledge of the process at step  $l$ ,  $P^-(l+1) = \langle (E(l+1) - \hat{E}^-(l+1))^2 \rangle$  is the *a priori* estimate of the covariance of  $E$ , and  $P(l) = \langle (E(l) - \hat{E}(l))^2 \rangle$  is the *a posteriori* estimate of the covariance.

The filter measurement update equations are:

$$\begin{aligned} K(l) &= \frac{P^-(l)}{(P^-(l) + R(l))}, \\ \hat{E}(l) &= \hat{E}^-(l) + K(l)(z(l) - \hat{E}^-(l)), \\ P(l) &= (1 - K(l))P^-(l). \end{aligned} \quad (15)$$

Note that, the Kalman filter assumes that  $w(l)$  and  $v(l)$  have normal distributions. However, statistical characteristics of the

tracking function output most likely will not be Gaussian since they are an outcome of a nonlinear process. In practice very few processes possess the noise terms that are normally distributed and, in case this deviation is due to some nonlinear measurement function, one usually tries to find inverse transform to get Gaussian variables. If such a transform can not be found, however, Kalman filtering is still used since it provides an accurate estimates of the first two moments of this distributions.

Finally, for each record, the damage tracking metric  $e_k$  and its variance are estimated as,

$$e_k = \langle \hat{E}(l) \rangle, \text{ and } \sigma_{e_k}^2 = \langle (\hat{E}(l) - e_k)^2 \rangle. \quad (16)$$

Following Eq. (5) and assuming linear observability,  $e_k$  is a linear function of  $\phi$ . Given a damage evolution law in terms of  $\phi$ , we can estimate the current state of damage based on estimates of  $e_k$  using an extended Kalman or some other nonlinear extension of the Kalman filter. Given a failure value (surface) of the damage variable, we can use the damage model and estimated current damage state to estimate the remaining time-to-failure.

### Nonlinear Recursive Estimation of Damage State

In this paper we assume that the form of the damage model is known from previous calculations or is determined *a posteriori*. The discrete-time measurement equation for  $z_\phi(k) \equiv e(k)$  is linear (following Eq. 5), but the damage process equation is usually nonlinear:

$$\begin{aligned} \phi(l+1) &= \phi(l) + \epsilon g(\phi(l))\Delta t + w_\phi(l+1), \\ z_\phi(l+1) &= C\phi(l+1) + c + v_\phi(l+1), \end{aligned} \quad (17)$$

where  $g$  is a given nonlinear function;  $\Delta t = Mt_s = t_D$  for consecutive data records with no gaps;  $C$  and  $c$  are scalar parameters introduced in Eq. (5); and the process (model) noise  $w_\phi(l)$  and measurement noise  $v_\phi(l)$  are assumed to be white, independent random variables with Gaussian distributions  $p(w_\phi(l)) \sim N(0, Q_\phi(l))$  and  $p(v_\phi(l)) \sim N(0, R_\phi(l))$ . Here, we take  $R_\phi(l) \propto \sigma_{e_k}^2(l)$  to be the covariance associated with each measurement  $z_\phi(l)$  and  $Q_\phi$  is an estimate of the damage model error.

Eqs. (17) are nonlinear and hence we have to use some nonlinear technique to estimate the state of  $\phi$ . It is common practice to use the extended Kalman filter for nonlinear systems. However, it is not suitable for systems with strong nonlinearities. Here, we use so-called Unscented filtering (Julier and Uhlmann, 1997). For completeness, we will summarize this general approach to nonlinear filtering as it applies to our problem. Let us consider a scalar random variable  $X(l)$  with mean  $\bar{X}(l)$  and variance  $P(l)$  for which we would like to predict the mean  $\bar{X}^-(l+1)$

and variance  $P^-(l+1)$  of random variable  $X(l+1)$  where random variable  $X(l+1)$  is related to  $X(l)$  by the nonlinear transformation  $X(l+1) = G(X(l))$ . Then the general procedure is:

1. Calculate the set of translated *sigma points*  $\{\mathcal{X}_i(l)\}$  from the  $P(l)$  as
$$\{\mathcal{X}_0(l), \mathcal{X}_1(l), \mathcal{X}_2(l)\} = \{\bar{X}(l), \bar{X}(l) - \sigma(l), \bar{X}(l) + \sigma(l)\}, \quad (18)$$
where,  $\sigma(l) = \sqrt{3P(l)}$ .
2. The transformed set of sigma points are evaluated by  $\mathcal{X}_i(l+1) = G(\mathcal{X}_i(l))$ .
3. Compute  $\bar{X}^-(l+1)$  and  $P^-(l+1)$  by computing the mean and covariance of the set  $\{\mathcal{X}_i(l+1)\}$ .

The filter measurement update equations are similar to Eqs. (15):

$$\begin{aligned} K_\phi(l) &= \frac{CP_\phi^-(l)}{(C^2P_\phi^-(l) + R_F(l))}, \\ \hat{\phi}(l) &= \hat{\phi}^-(l) + K_\phi(l)(z_\phi(l) - C\hat{\phi}^-(l)), \\ P_\phi(l) &= (1 - CK_\phi(l))P_\phi^-(l). \end{aligned} \quad (19)$$

### Linear Kalman Filter for Time to Failure Estimation

We can design a nonlinear recursive filter for time to failure estimation based on the damage model and damage tracking metric output, as described in the previous section. However, since we already have a damage state estimate, we use it for time to failure estimation. Using the damage evolution model (as defined by  $g$  in Eq. 17), we can derive an expression for the remaining useful life  $t_F$ , given a predefined failure surface (value)  $\phi(t+t_F) = \phi_F$ :  $t_F(l) = z_F(\phi(l), \phi_F)$ . Then, for the time to failure estimation module we use the following discrete-time state transition and measurement equations

$$\begin{aligned} t_F(l+1) &= t_F(l) - \Delta t + w_F(l+1), \\ z_F(l+1) &= t_F(l+1) + v_F(l+1), \end{aligned} \quad (20)$$

where,  $\Delta t = Mt_s$ , again, for consecutive data records with no gaps between them;  $w_F(l)$  and  $v_F(l)$  are assumed to be white, independent random variable with Gaussian distributions  $p(w_F(l)) \sim N(0, Q_F(l))$  and  $p(v_F(l)) \sim N(0, R_F(l))$ , respectively. In this instance, we take  $R_F(l) \propto \frac{dz_F(\phi(l), \phi_F)}{d\phi} \sigma_\phi^2(l)$  to be the covariance associated with each measurement  $z_F(l)$ , where  $\sigma_\phi^2(l)$  is a posteriori covariance for estimate of  $\phi(l)$ . Since the Eqs. (27) are linear we can again use the linear Kalman filter, in this case with time update equations:

$$\begin{aligned} \hat{t}_F^-(l+1) &= \hat{t}_F(l) - \Delta t, \\ P_F^-(l+1) &= P_F(l) + Q_F(l). \end{aligned} \quad (21)$$

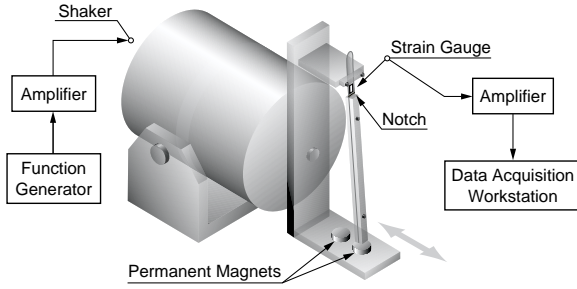


Figure 2. Schematic diagram of the modified experimental apparatus and the two-well magneto-mechanical oscillator.

The corresponding measurement update equations are:

$$\begin{aligned}
 K_F(l) &= \frac{P_F^-(l)}{(P_F^-(l) + R_F(l))}, \\
 \hat{t}_F(l) &= \hat{t}_F^-(l) + K_F(l)(z_F(l) - \hat{t}_F^-(l)), \\
 P_F(l) &= (1 - K_F(l))P_F^-(l).
 \end{aligned} \tag{22}$$

## EXPERIMENTAL APPLICATION

The experimental system is a modification the standard two-well magneto-mechanical oscillator (Moon and Holmes, 1979). The system is a cantilever beam with stiffeners, and with two magnets near its end providing two-well potential (see Fig. 2). A shallow notch is filed in the beam below where the strain gauge is placed, just above the stiffeners. The system is mounted on a shaker and is forced at about 6 Hz. The forcing amplitude was set to obtain chaotic output. The crack in the beam slowly propagated starting from the notch for about 2.5 hours till complete fracture. Strain gauge output is sampled at 100 Hz sampling frequency ( $t_s = 0.01$  sec), digitized (12 bit A/D), and stored on a computer.

Delay time and embedding dimension were estimated to be  $5t_s$  and 5, respectively. The first  $2^{15}$  data points were used for the reference data set, and  $N = 16$  nearest neighbors were used for the local linear model parameter estimation. The estimated average poinwise dimension of the reference data set was  $d_f = 2.86$ . Throughout the experiment the system underwent many bifurcations causing repeated periodic/chaotic transitions. Thus, loading on the crack was irregular and most of the time chaotic.

After going through the embedding and modeling process we estimated the tracking function  $e_5$  by calculating the short-time prediction error  $\mathbf{E}_5(l)$  of the reference model along with the distance  $r_l$  between the point  $\mathbf{y}(l)$  to the farthest of nearest neighbor points  $\mathbf{y}^r(l)$  used in modeling. A sample distribution of  $|E_5(l)|$  is highly asymmetrical and is shown on Fig. 3 (top). However, it can be approximated by a lognormal distribu-

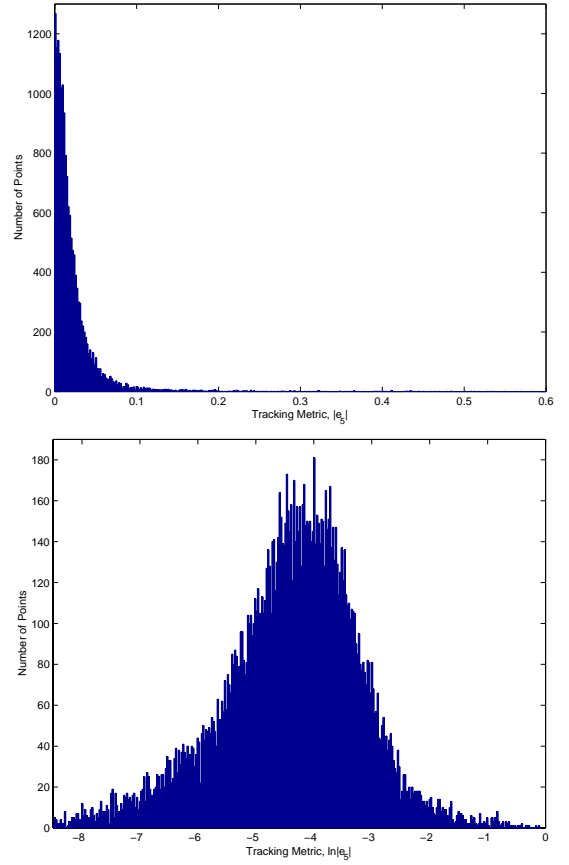


Figure 3. Distribution of  $(|E_5|)$  (top) and distribution of  $\log(|E_5|)$  (bottom).  $2^{12}$  points were used for the histogram starting from  $2^{16}$ -th point.

tion. Indeed, if we take a natural log of absolute value of tracking function we get an approximately Gaussian distribution (Fig. 3 bottom). Thus, to calculate the tracking function  $e_5$  we apply Kalman filter Eqs. (14)–(15) to  $\ln(\|\mathbf{E}_5(l)\|)$  and transform it back using exponential function. Here, we used  $R(l) = 10^3 r_l^{d_f}$ ,  $Q = 10^{-5}$ , and  $P^-(1) = 1$ . Consecutive data records of  $M = 2^{12}$  points (with no overlapping sections or gaps between records) were used to calculate the mean and standard deviation in estimates Eq. (16).

In this paper we assume that the form of empirical or analytical damage evolution law is known *a priori*. In our particular case, in prior experiments it was observed that the reciprocal of estimated tracking metric  $e_5$  was a linear function of time. This observation is consistent with a power law model of damage evolution:

$$\frac{d\phi}{dt} = \epsilon\phi^\alpha, \tag{23}$$

where the solution to this equation for an arbitrary constant  $\beta$

$$\phi^{1-\alpha} = (\epsilon t + \beta)(1 - \alpha), \quad (24)$$

gives a linear relation between  $1/\phi$  and  $t$  for a parameter  $\alpha = 2$ . Further, fitting a straight line to this trend we also estimated the rate constant  $\epsilon = 39.06/t_T$ .

Since we do not have any independent measurement of damage state to determine parameters of Eq. (5), we assume  $C = 1$  and  $c = 0$  for linear measurement equation in Eqs. (17) corresponding to  $z(k) = e(k)$ . Thus, the discrete-time damage process and measurement equations are:

$$\begin{aligned} \phi(l+1) &= \phi(l) + \Delta t \epsilon \phi(l)^\alpha + w_\phi(l+1), \\ z_\phi(l+1) &= \phi(l+1) + v_\phi(l+1). \end{aligned} \quad (25)$$

The results of state estimation using unscented filter and Eq. (19) are shown in Fig. 4 (top). Here, we take  $R_\phi(l) = 10^2 \sigma_{e_s}^2(l)$ ,  $Q_\phi = 10^{-2}$ , and  $P_\phi^-(1) = 1$ .

Using Eq. (24) we can derive expression for the remaining useful life  $t_F$ , given predefined failure surface (value)  $\phi(t + t_F) = \phi_F$ ,

$$t_F = \frac{\phi_F^{1-\alpha} - \phi^{1-\alpha}}{\epsilon(1-\alpha)}. \quad (26)$$

Then, for the time to failure estimation module we have the following discrete-time state transition and measurement equations

$$\begin{aligned} t_F(l+1) &= t_F(l) - \Delta t + w_F(l+1), \\ z_F(l+1) &= t_F(l+1) + v_F(l+1), \end{aligned} \quad (27)$$

The results of time-to-failure prediction using Eqs. (21)–(22) are shown in Fig. 4 (bottom). For this calculation, we have used  $Q_F = 10^{-4}$ ,  $P_F^-(1) = 10^2$ , and  $\phi_F = 0.13$ .

Fig. 4 clearly shows a power law trend in the estimated damage state evolution. Standard deviations for each estimate were too small to resolve in the plot. For fixed  $\phi_F$  and using initial guess calculated by Eq. (26), the failure prediction algorithm was able to accurately converge to the *a posteriori* known time-to-failure line well in advance of the actual fracture in the beam.

## CONCLUSIONS

In this paper we have presented a new algorithm for estimating the damage state and predicting failures in mechanical systems. The algorithm is general purpose, since it is not tied

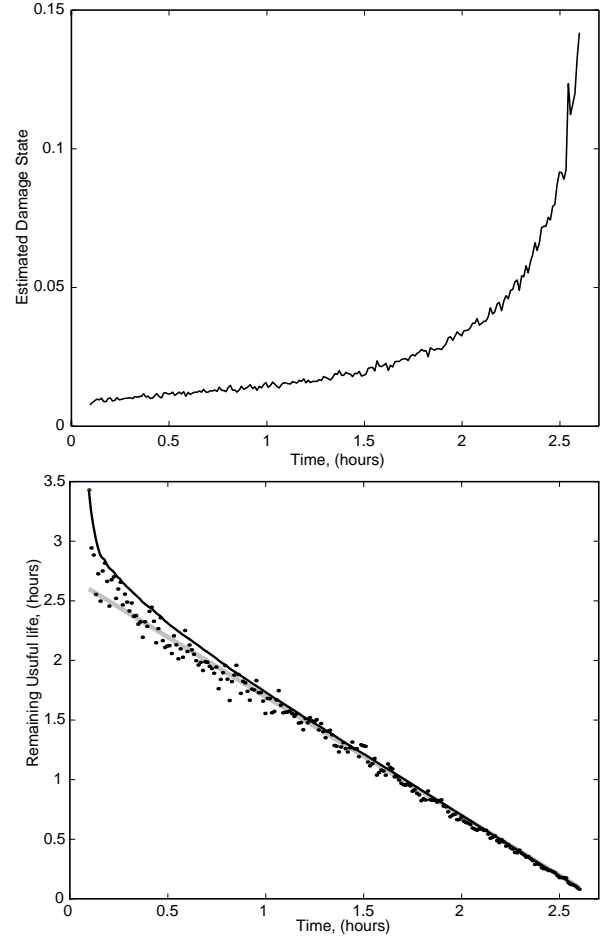


Figure 4. Damage state estimation and failure prediction for a nonlinear vibrating beam with growing crack: (top) damage state estimation showing power law behavior; (bottom) time-to-failure predictions, made using two-time-scale recursive estimator/predictor, starting from an initial guess. In the time-to-failure predictions, the thick solid grey line indicates the true time to failure (known a posteriori).

to any specific damage physics. It assumes that the system possesses time scale separation, i.e., damage process occurs on a much slower time scale than the observable dynamics of a system. It is assumed implicitly that the fast subsystem is governed by some ordinary differential equation Eq. (1a). However, for a failure prediction algorithm only a mathematical model for Eq. (1b) is required.

We gave a description of the three major parts of the diagnostics/prognostics algorithm. In the first part, reference model construction and short-time prediction error statistics were described. In the second part, we discussed how prediction error statistics are used to estimate the change in the damage state of the system. And finally, in the third part, we gave a description

of a prognostics module.

The algorithm was applied to a magneto-mechanical oscillator with a growing crack in the beam. As the crack grew, the system underwent many bifurcations causing repeated periodic/chaotic transitions. This resulted in a very complicated strain time series. Nevertheless, the method used here smoothly tracked the accumulating damage. The results demonstrated that our method is capable of predicting the failure time of a beam well in advance of the actual fracture.

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