

Procedure for tracking damage evolution and predicting remaining useful life with application to an electromechanical experimental system

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ABSTRACT

A general method for tracking the evolution of hidden damage processes and predicting remaining useful life is presented and applied experimentally to an electromechanical system with a failing supply battery. The fundamental theory for the method is presented. In this theory, damage processes are viewed as occurring in a hierarchical dynamical system consisting of "fast", directly observable subsystem coupled with a "slow", hidden subsystem describing damage evolution. In the algorithm, damage tracking is achieved using a two-time-scale modeling strategy based on phase space reconstruction. Using the reconstructed phase space of the reference (undamaged) system, short-time predictive models are constructed. Fast-time data from later stages of damage evolution of a given system are collected and used to estimate the short time reference model prediction error or a tracking metric. The tracking metric is used as an input to a nonlinear recursive filter, the output of which provides an estimate of the current damage (or, equivalently, health) state. Estimates of remaining useful life (or, equivalently, time to failure) are obtained recursively using the current damage state estimates under the assumption of a particular battery voltage evolution model. In the experimental application, the method is shown to accurately estimate both the battery state and the time to failure throughout the whole experiment.

Keywords: condition monitoring, damage tracking, failure prediction, machinery diagnostics and prognostics

1. INTRODUCTION

Machinery components and their parts invariably acquire defects during their operation, maintenance, or production. Under certain forcing, operational or environmental conditions some of these defects can initiate damage processes in systems that eventually lead to failures. Any type of failure is undesirable, however, failures in critical system components can bring devastating results. In today's highly competitive marketplace, both military and civil industries strive to minimize their capital and operational costs by trying to utilize the full life cycle of their machinery without sacrificing human, production, or environmental safety. Condition based maintenance and failure prognostics technologies are addressing this issue. However, the development of these technologies is hampered by the fact that there is very little known about various types of damage physics. For example, in fracture mechanics, the time required to initiate a crack is considered to be unpredictable for fatigue damage, which is one of the major life-limiting factors for most structural components. Even when the physics of a damage process is known, i.e. we have suitable damage evolution model, access to the damage state variable is usually limited due to the "hidden" nature of damage. In some cases, after removing machinery from operation for closer inspection, one can determine the current state of damage. However, this procedure results in loss of productivity and higher cost of maintenance. Therefore, there is a clear need for methods that allow damage state estimation based on the measurements taken from machinery in operation.

For true on-line machinery failure prognostics one needs systems that continually estimate the current state of damage variables. Then, given a damage evolution law and its parameters, one can estimate time to failure. Due to the problems mentioned above, even when experimental systems have the same type of damage physics and, hence, the same model structure, we can not expect their respective model parameters to be the same. Therefore, for true

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general purpose prognostics, the ability to estimate damage model parameters on-line in real time is necessary. The problem of simultaneous damage state and model parameter estimation is a subject of separate research. In this paper we assume that the form of the damage model and its parameters are known *a priori*.

Most of the earlier research on the subject of damage (failure) detection and identification was concentrated on the development of heuristic methods based on time or frequency domain signal processing techniques.¹⁻⁴ More recent research focuses on heuristic methods that can utilize both time and frequency domain information.⁵⁻⁷ In both cases, these are mainly failure detection methods, i.e. the main emphasis is on the development of a feature vector that will indicate when the system parameters have reached some preset failure values. The main advantage of these methods is that they are easy to implement and sometimes work very well. However, there is no theoretical basis to determine *a priori* if a given method is going to work well for a particular system without prior experimentation or knowledge.

Model based methods, in contrast, overcome some of the limitations of heuristic methods at the expense of more complex development and higher implementation costs.⁸⁻¹⁰ They are general in the sense that if some properties of the system or damage physics change, models can be readjusted to accommodate the change. Their main advantage is that knowing a model structure gives the ability to tie the changes in feature vectors to model parameter changes.¹¹ However, in many cases one does not know the appropriate model for the system, and since most damage processes are nonlinear, model development costs are high. This problem is addressed by the methods that use neural networks,¹²⁻¹⁴ genetic algorithms,¹⁵ and other data-based modelling techniques to replace physics based models. However, in this approach we lose the ability to directly tie damage evolution to changes in a system's physical parameters. It should be mentioned that most of the model based schemes, just like most heuristic methods, are used primarily as damage *detection* methods.

In this paper we present a general damage state *estimation* and failure prognostics method that overcomes all the above mentioned limitations at the expense of an abstract formulation of the problem in phase space. The method presented here is an extension of a previously presented experimental method for damage evolution tracking.^{16,17} In this approach, we implicitly treat the hidden damage process as evolving in a hierarchical dynamical system consisting of a "fast-time" subsystem coupled to a "slow-time" subsystem with the form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\mu}(\boldsymbol{\phi}), t), \quad (1a)$$

$$\dot{\boldsymbol{\phi}} = \epsilon \mathbf{g}(\mathbf{x}, \boldsymbol{\phi}, t), \quad (1b)$$

where: $\mathbf{x} \in \mathbb{R}^n$ is the fast dynamic variable (we assume that we can measure some functional combination of its components); $\boldsymbol{\phi} \in \mathbb{R}^m$ is the slow dynamic variable ("hidden" damage state); the parameter vector $\boldsymbol{\mu} \in \mathbb{R}^p$ is a function of $\boldsymbol{\phi}$; t is time; and $0 < \epsilon \ll 1$ defines the time scale separation between the fast dynamics and slow drift. If ϵ were exactly zero, then $\boldsymbol{\mu}(\boldsymbol{\phi}_0) = \boldsymbol{\mu}_0$ would be a constant vector of parameters in Eq. (1a); since ϵ is very small, we might consider Eq. (1a) to be a model for a system with slowly drifting parameters.

The method is appropriate for systems where hidden processes evolve on a much slower time scale than the observable dynamics that can be measured on-line in real time. This time scale separation is common for dynamical systems with evolving damage processes. For example, crack growth in a spinning shaft can be characterized by a time scale of hours, days, weeks or even years, while a time scale of shaft vibration signature is characterized by milliseconds or seconds.

In the next section of this paper we present the damage tracking algorithm using phase space reconstruction. In Section 3, we describe the damage state estimation and failure prognosis procedure that uses the tracking algorithm output. The experimental application of the method to an electro-mechanical system is presented in Section 4. Finally, in Section 5, we conclude our paper by summarizing the main features of the method and discussing the experimental results.

2. DAMAGE TRACKING ALGORITHM

The essential idea of tracking can be motivated as follows. Consider the system of Eqs. (1) with a fixed parameter vector $\boldsymbol{\mu}$ and a fixed damage state $\boldsymbol{\phi}$ (i.e., $\epsilon = 0$). Then, for some initial time coordinate t_0 and some initial space coordinate $\mathbf{x}(t_0) = \mathbf{x}_0$, at time $t_0 + t_p$ we have:

$$\mathbf{x}(t_0 + t_p) = \mathbf{X}(\mathbf{x}_0, t_0, t_p; \boldsymbol{\mu}(\boldsymbol{\phi})). \quad (2)$$

If we fix t_0 , t_p and \mathbf{x}_0 , then \mathbf{X} is purely a function of $\boldsymbol{\mu}(\phi)$ that describes a p -parameter surface in \mathbb{R}^n space.

In principle, the value of $\mathbf{X}(\boldsymbol{\mu}(\phi))$ can be evaluated for every ϕ , and given certain yet to be determined observability conditions one can construct mappings $\mathbf{X}^{-1}(\boldsymbol{\mu}) : \mathbb{R}^n \rightarrow \mathbb{R}^p$ or $\mathbf{X}^{-1}(\phi) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ required for true tracking. Therefore, if the function $\mathbf{X}(\boldsymbol{\mu}(\phi))$ can be determined experimentally, subsequently it can be used to determine $\boldsymbol{\mu}(\phi)$ or ϕ provided the observability conditions are satisfied.

2.1. Damage Tracking Function

Let us consider the case when $\epsilon \neq 0$. For $0 < \epsilon \ll 1$, ϕ cannot be treated as a parameter and, in this case, the complete state of the system Eqs. (1) is the direct product (\mathbf{x}, ϕ) . However, for $t_p \ll 1/\epsilon$ we can assume that $\phi = \phi_0 + \mathcal{O}(\epsilon)$, where $\phi_0 = \phi(t_0)$. Now, the state of the fast subsystem at time $t_0 + t_p$ can be written as follows:

$$\begin{aligned} \mathbf{x}(t_0 + t_p; \epsilon) &= \mathbf{X}(\mathbf{x}_0, \boldsymbol{\mu}(\phi_0), t_0, t_p), \\ &= \mathbf{X}(\mathbf{x}_0, t_0, t_p; \boldsymbol{\mu}(\phi_0)) + \mathcal{O}(\epsilon), \\ &= \mathbf{X}(\mathbf{x}_0, t_0, t_p; \boldsymbol{\mu}(\phi_R)) + \frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi} (\phi_0 - \phi_R) + \mathcal{O}(\|\phi_0 - \phi_R\|^2) + \mathcal{O}(\epsilon), \end{aligned} \quad (3)$$

where on the third line we have Taylor expanded the expression about $\phi_R = \phi(t_R)$, the reference (healthy) value of the damage state variable for some reference time $t_0 = t_R$. Then, for fixed coordinates \mathbf{x}_0 , and ϕ_R , and fixed t_p , we define a *tracking function* as follows:

$$\begin{aligned} \mathbf{e} &\equiv \mathbf{X}(\mathbf{x}_0, \boldsymbol{\mu}(\phi_0), t_0, t_p) - \mathbf{X}(\mathbf{x}_0, \boldsymbol{\mu}(\phi_R), t_0, t_p), \\ &= \frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi} (\phi_0 - \phi_R) + \mathcal{O}(\|\phi_0 - \phi_R\|^2) + \mathcal{O}(\epsilon), \end{aligned} \quad (4)$$

where the coefficient matrices in the Taylor expansion are evaluated with the initial conditions $\mathbf{x}(t_0) = \mathbf{x}_0$, and ϕ_R .

Now, it is clear that the observability condition is that the matrix $\frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi}$ must have maximal rank. That is, we should have $n > p > m$, in addition to $\text{Range}(\frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}}) \in \mathbb{R}^n$ and $\text{Range}(\frac{\partial \boldsymbol{\mu}}{\partial \phi}) \in \mathbb{R}^p$ having dimensions p and m , respectively.

Assuming that the linear operator in the first term of the Taylor expansion Eq. (4) has maximal rank, we expect the output of the tracking function to be an affine transformation of the damage variable (*linear observability*):

$$\mathbf{e} = \mathbf{C}(\mathbf{x}_0, t_0, t_p) \phi_0 + \mathbf{c}(\mathbf{x}_0, t_0, t_p, \epsilon), \quad (5)$$

where, $\mathbf{C} = \frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi}$ is an $n \times m$ matrix, and $\mathbf{c} = -\frac{\partial \mathbf{X}}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial \phi} \phi_R + \mathcal{O}(\|\phi_0 - \phi_R\|^2) + \mathcal{O}(\epsilon)$ is an $n \times 1$ vector; all terms are evaluated using the initial conditions $\mathbf{x}(t_0) = \mathbf{x}_0$, and ϕ_R .

Now, in principle, one can repeatedly start the fast subsystem from the same initial coordinates t_0 and \mathbf{x}_0 at different stages of damage evolution, and record corresponding \mathbf{e} for some fixed t_p . Thus, assuming linear observability, one could estimate the corresponding damage state by inverting the affine transformation described by Eq. (5). In an experimental context, however, this procedure is prone to errors. The tracking quality or sensitivity (i.e., $\|\mathbf{C}(\mathbf{x}_0, t_0, t_p)\|$, in our case) is a function of phase space coordinates. To deal with this variability and to improve robustness of the method, one can examine the tracking function for many different values \mathbf{x}_0 or t_0 and come up with some average measure of \mathbf{e} over a set of given initial conditions. Such an approach, where the behavior of two systems is studied for many different values of starting times and initial conditions is philosophically similar to a model-based approach.

It is usually impossible to repeatedly start the system from the same initial condition. However, we can use data or physics based models for Eqs. (1a) to approximate system's flow starting from particular initial condition. To actually calculate the tracking function \mathbf{e} , we need to know how the fast subsystem evolves over the time interval t_p for the current value of ϕ , as well as how the subsystem *would have* evolved for the reference value of ϕ_R , both evolving from same initial condition \mathbf{x}_0 . Since the system's fast time behavior for the current damage state, characterized by $\phi = \phi_0 + \mathcal{O}(\epsilon)$, is directly measurable (i.e., we can reconstruct fast dynamics using a sensor measurement from the fast subsystem), the strategy used in this paper is to compare it to the predictions of a data based *reference model* describing the fast subsystem behavior for the $\phi = \phi_R + \mathcal{O}(\epsilon)$.

2.2. Reference Local Linear Models

By design, the system considered in this paper exhibits chaotic motion in its reference state. This was done to exploit the well-known fundamental property of chaotic attractors of exploring a fairly large region of phase space, thus making recorded chaotic trajectories useful for data-based modeling. It should be emphasized, however, that this use of chaos to obtain the reconstructed reference phase space should be viewed as an experimental and theoretical convenience: in principle, the same tracking procedure described in the following sections could be applied to study arbitrary global behaviors (including multiple coexisting periodic orbits and random excitation) for a variety of systems, both linear and nonlinear.

For clarity of discussion, we have to specify what kind of data is needed for the algorithm. We assume that scalar time series measured from the fast subsystem Eq. (1a) are sampled with sampling time t_s and collected in records of size $t_D = Mt_s$. The total data collection time t_T does not necessarily equal $N_T t_D$, where N_T is the total number of data records: $t_T \geq N_T t_D \gg t_s$ since there may be gaps between the data records. We also assume that the rate of damage accumulation is very slow, so that, in particular, $t_D \ll 1/\epsilon$. Thus, the total change in the damage variable ϕ is very small over any data collection time span t_D and so for any $t_0 \in t_D$ we can write $\phi = \phi_0 + \mathcal{O}(\epsilon) \approx \phi_0$. Thus, given this assumption of quasistationarity over any single data collection interval, in what follows we drop the subscript zero from ϕ .

Using delay reconstruction^{18,19} the recorded scalar time series $\{x(n)\}_{n=1}^M$ is used to reconstruct a phase space of appropriate dimension (say, d) for the system. In this reconstructed phase space the scalar time series is converted to a series of vectors $\mathbf{y}^T(n) = (x(n), x(n+\tau), \dots, x(n+(d-1)\tau)) \in \mathbb{R}^d$, where τ is a suitable delay parameter. Embedding parameters τ and d are determined using the first minimum of the average mutual information²⁰ and the false nearest neighbors²¹ method, respectively.

The reconstructed state vectors are governed by an as yet undetermined map of the form $\mathbf{y}(n+1) = \mathbf{P}(\mathbf{y}(n); \phi)$. The observation period t_p discussed earlier now corresponds to picking an integer k ($t_p = kt_s$). We examine the k^{th} iterate of the map \mathbf{P} , i.e.,

$$\mathbf{y}(n+k) = \mathbf{P}^k(\mathbf{y}(n); \phi). \quad (6)$$

Working in the global reconstructed phase space of the dynamical system, local models which show how neighborhoods about each data point are mapped forward in time can be estimated. The simplest such model is a local *linear* model relating the state at time n , $\mathbf{y}(n)$ to the state $\mathbf{y}(n+k)$ at time $n+k$:

$$\mathbf{y}(n+k) = \mathbf{A}_n \mathbf{y}(n) + \mathbf{a}_n = \mathbf{B}_n \hat{\mathbf{y}}(n), \quad (7)$$

where the model parameter matrix \mathbf{A}_n and parameter vector \mathbf{a}_n are determined by regression at each point in the data set, and where

$$\hat{\mathbf{y}}(n) = [\mathbf{y}^T(n), 1]^T, \text{ and } \mathbf{B}_n = [\mathbf{A}_n \mid \mathbf{a}_n]. \quad (8)$$

Note that \mathbf{A}_n and \mathbf{a}_n provide local approximations to \mathbf{P}^k , and hence they will in general depend on k .

2.3. Estimating the Tracking Function

The phase space geometry of the tracking function estimation problem is shown schematically in Fig. 1. First, we collect a reference data set and reconstruct the reference phase space (reference trajectories are shown in gray). Then, at some later time we experimentally obtain a number of points $\mathbf{y}(l)$ ($l = 1, 2, \dots$), and the corresponding points after k steps, $\mathbf{y}(l+k)$, for the “changed” system. We look up N points in the reference set that are nearest neighbors of each point $\mathbf{y}(l)$ and the corresponding N reference data points k steps later. Based on these points, we build the reference model for each point $\mathbf{y}(l)$ as described in the previous section. In the reference system the same initial points $\mathbf{y}(l)$ would have been mapped to points $\mathbf{y}^R(l+k)$. Since we have an imperfect reference model, it actually maps these same points to some other points $\mathbf{y}^M(l+k)$.

We introduce the following error vectors (for clarity, the dependence of the \mathbf{E} 's on l is suppressed in the figure):

$$\mathbf{E}_k^R(l) := \mathbf{y}(l+k) - \mathbf{y}^R(l+k), \quad \text{the true error,} \quad (9a)$$

$$\mathbf{E}_k(l) := \mathbf{y}(l+k) - \mathbf{y}^M(l+k), \quad \text{the measured error,} \quad (9b)$$

$$\mathbf{E}_k^M(l) := \mathbf{y}^M(l+k) - \mathbf{y}^R(l+k), \quad \text{the modeling error.} \quad (9c)$$

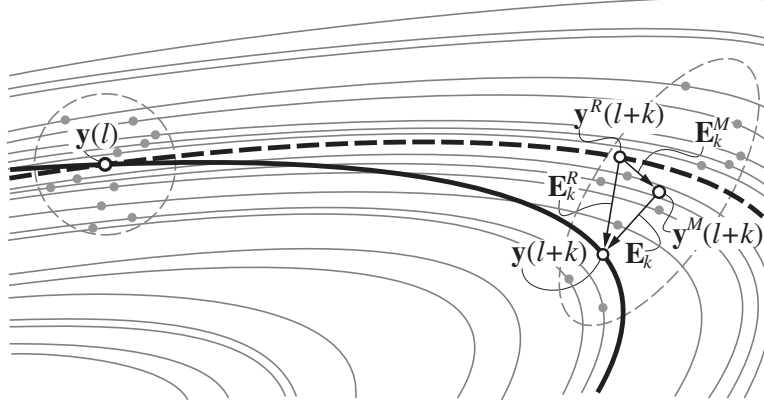


Figure 1. Schematic drawing illustrating the basic ideas about estimating tracking function. Trajectories of the reference system in phase space are shown in gray. The solid black line represents a measured trajectory of the perturbed system, passing through $\mathbf{y}(l)$, and the dashed gray line represents where a trajectory would have gone in the reference system, if starting from the same point $\mathbf{y}(l)$.

The tracking function (Eq. 4) for initial point $\mathbf{y}(l)$ in the reconstructed phase space can be estimated as

$$\mathbf{e}_k(l) = \mathbf{P}^k(\mathbf{y}(l), \phi) - \mathbf{P}^k(\mathbf{y}(l), \phi_R) = \mathbf{E}_k^R(l) = \mathbf{E}_k(l) + \mathcal{O}(\mathbf{E}_k^M(l)). \quad (10)$$

We are interested in estimating the slow damage variable ϕ , which requires an estimate of $\mathbf{e} = \mathbf{E}_k^R$. However, we can only measure the error $\mathbf{E}_k(l)$. Thus, what is needed is a method for estimating the true error \mathbf{E}_k^R given only $\mathbf{E}_k(l)$. Furthermore, the above equation uses the information at only one point in the phase space (as indicated by the index l), whereas we wish to use all of the data available to us within one data record. Both of these issues can be addressed by an appropriate filter, which is the subject of the next subsection.

2.4. Damage Tracking Metric

In this paper we focus on scalar damage variables. Therefore, for the actual scalar *estimated tracking function* we use $\|\mathbf{E}_k^R(l)\|$. However, to use the data from an entire data record, we consider instead the *tracking metric*

$$e_k = \langle \|\mathbf{E}_k^R(l)\| \rangle, \quad (11)$$

where $\langle \odot \rangle$ represents *root mean square* (RMS) value of \odot over an entire data record of time span t_D .

We can then attempt to estimate $\|\mathbf{E}_k^R(l)\|$ using only the measurable $\|\mathbf{E}_k(l)\|$ time series with an appropriate filter \mathcal{F} , so that Eq. (11) becomes:

$$e_k = \langle \mathcal{F}(\|\mathbf{E}_k(l)\|) \rangle. \quad (12)$$

One way to design the filter \mathcal{F} would be to make use of an explicit analytical model for the fast subsystem Eq. (1a), which could be used to derive a model describing the dynamics of $\|\mathbf{E}_k(l)\|$. In our case, we have assumed that such a model is not available. However, considering that the fast sampling time $t_s \ll t_D \ll 1/\epsilon$, a constant model should provide an acceptable approximation for the tracking function evolution over the sampling time interval t_s . That is, over the relatively short sampling time, the best estimate of the future “error state,” $\|\mathbf{E}_k^R(l+1)\|$, is its current value, $\|\mathbf{E}_k^R(l)\|$, which is, obviously, a linear relationship.

For linear systems the Kalman filter²² provides the optimal minimum mean square error estimator, that can be calculated recursively. Specifically, for \mathcal{F} we consider Kalman filter designed for random constant parameter estimation. The linear process difference equation for the scalar tracking function $E(l) \equiv \|\mathbf{E}_k^R(l)\|$ with a scalar measurement function $z(l) \equiv \|\mathbf{E}_k(l)\|$ is:

$$\begin{aligned} E(l+1) &= E(l) + w(l+1), \\ z(l+1) &= E(l+1) + v(l+1), \end{aligned} \quad (13)$$

where the process (model) noise $w(l)$ and measurement noise $v(l)$ are assumed to be white, independent random variables with Gaussian distributions ($p(w) \sim N(0, Q)$ and $p(v(l)) \sim N(0, R(l))$).

In this work, the constant Q corresponds to the average amplitude of fluctuations in $\|\mathbf{E}_k^R(l)\|$ as the location in phase space changes along a given trajectory. In addition, $R(l)$ corresponds to fluctuations in $\|\mathbf{E}_k^R(l)\|$ due to changes in the local linear model accuracy from point to point in phase space. We take $R(l) \propto r_l^{d_f}$, where r_l is the distance from the point $\mathbf{y}(l)$ to the farthest of all N nearest neighbors used for local modeling, and d_f is the estimated dimension of the data in the reference phase space, i.e., average pointwise dimension (see Ott, 1993, pg. 86). The argument behind this choice is that the accuracy of the local linear models is proportional to the generalized volume occupied by the fixed number of nearest neighbors, and that the volume by itself scales like the r_l raised into the d_f power.

The tracking function filter time update equations are:

$$\begin{aligned}\hat{E}^-(l+1) &= \hat{E}(l), \\ P^-(l+1) &= P(l) + Q.\end{aligned}\tag{14}$$

where, $\hat{E}^-(l+1)$ is the *a priori* estimate of $E(l+1)$ given the knowledge of the process at step l , $P^-(l+1) = \langle (E(l+1) - \hat{E}^-(l+1))^2 \rangle$ is the *a priori* estimate of the covariance of E , and $P(l) = \langle (E(l) - \hat{E}(l))^2 \rangle$ is the *a posteriori* estimate of the covariance.

The filter measurement update equations are:

$$\begin{aligned}K(l) &= \frac{P^-(l)}{(P^-(l) + R(l))}, \\ \hat{E}(l) &= \hat{E}^-(l) + K(l)(z(l) - \hat{E}^-(l)), \\ P(l) &= (1 - K(l))P^-(l).\end{aligned}\tag{15}$$

Note that, the Kalman filter assumes that $w(l)$ and $v(l)$ have normal distributions. However, statistical characteristics of the tracking function output most likely will not be Gaussian since they are an outcome of a nonlinear process. In practice very few processes possess the noise terms that are normally distributed and, in case this deviation is due to some nonlinear measurement function, one usually tries to find inverse transform to get Gaussian variables. If such a transform can not be found, however, Kalman filtering is still used since it provides an accurate estimates of the first two moments of this distributions.

Finally, for each record, the damage tracking metric e_k and its variance are estimated as,

$$e_k = \langle \hat{E}(l) \rangle, \text{ and } \sigma_{e_k}^2 = \langle (\hat{E}(l) - e_k)^2 \rangle.\tag{16}$$

3. DAMAGE STATE ESTIMATION AND FAILURE PROGNOSTICS

Following Eq. (5) and assuming linear observability, e_k is a linear function of ϕ . Given a damage evolution law in terms of ϕ , we can estimate the current state of damage based on estimates of e_k using an extended Kalman or some other nonlinear extension of the Kalman filter. If only the general form of the damage law is known and its parameters are unknown, one can try to estimate them recursively along with the damage state. Given a failure value (surface) of the damage variable, we can use the damage model and estimated current damage state to estimate the remaining time-to-failure.

3.1. Nonlinear Recursive Estimation of Damage State

In this paper we assume that the form of the damage model is known from previous calculations or is determined *a posteriori*. The measurement equation for $z_\phi(k) \equiv e(k)$ is linear (following Eq. 5), but the damage process equation is usually nonlinear:

$$\begin{aligned}\phi(l+1) &= \phi(l) + \epsilon g(\phi(l))\Delta t + w_\phi(l+1), \\ z_\phi(l+1) &= C\phi(l+1) + c + v_\phi(l+1),\end{aligned}\tag{17}$$

where g is a given nonlinear function; $\Delta t = Mt_s = t_D$ for consecutive data records with no gaps; C and c are scalar parameters; and the process (model) noise $w_\phi(l)$ and measurement noise $v_\phi(l)$ are assumed to be white,

independent random variables with Gaussian distributions ($p(w_\phi(l)) \sim N(0, Q_\phi(l))$ and $p(v_\phi(l)) \sim N(0, R_\phi(l))$). Here, we take $R_\phi(l) \propto \sigma_{e_k}^2(l)$ to be the covariance associated with each measurement $z_\phi(l)$ and Q_ϕ is an estimate of the damage model error.

Eqs. (17) are nonlinear and hence we have to use some nonlinear technique to estimate the state of ϕ . It is common practice to use the extended Kalman filter for nonlinear systems. However, it is not suitable for systems with strong nonlinearities. Here, we use so-called Unscented filtering.²³ For completeness, we will summarize this general approach to nonlinear filtering as it applies to our problem. Let us consider a scalar random variable $X(l)$ with mean $\bar{X}(l)$ and variance $P(l)$ for which we would like to predict the mean $\bar{X}^-(l+1)$ and variance $P^-(l+1)$ of random variable $X(l+1)$ where random variable $X(l+1)$ is related to $X(l)$ by the nonlinear transformation $X(l+1) = G(X(l))$. Then the general procedure is:

1. Calculate the set of translated *sigma points* $\{\mathcal{X}_i(l)\}$ from the $P(l)$ as

$$\{\mathcal{X}_0(l), \mathcal{X}_1(l), \mathcal{X}_2(l)\} = \{\bar{X}(l), \bar{X}(l) - \sigma(l), \bar{X}(l) + \sigma(l)\}, \quad (18)$$

where, $\sigma(l) = \sqrt{3P(l)}$.

2. The transformed set of sigma points are evaluated by $\mathcal{X}_i(l+1) = G(\mathcal{X}_i(l))$.
3. Compute $\bar{X}^-(l+1)$ and $P^-(l+1)$ by computing the mean and covariance of the set $\{\mathcal{X}_i(l+1)\}$.

The filter measurement update equations are similar to Eqs. (15):

$$\begin{aligned} K_\phi(l) &= \frac{CP_\phi^-(l)}{(C^2P_\phi^-(l) + R_F(l))}, \\ \hat{\phi}(l) &= \hat{\phi}^-(l) + K_\phi(l)(z_\phi(l) - C\hat{\phi}^-(l)), \\ P_\phi(l) &= (1 - CK_\phi(l))P_\phi^-(l). \end{aligned} \quad (19)$$

3.2. Linear Kalman Filter for Time to Failure Estimation

We can design a nonlinear recursive filter for time to failure estimation based on the damage model and damage tracking metric output, as described in the previous section. However, since we already have a damage state estimate, we use it for time to failure estimation. Using the damage evolution model (as defined by g in Eq. 17), we can derive an expression for the remaining useful life t_F , given a predefined failure surface (value) $\phi(t_F) = \phi_F$: $t_F(l) = z_F(\phi(l), \phi_F)$. Then, for the time to failure estimation module we use the following discrete-time state transition and measurement equations

$$\begin{aligned} t_F(l+1) &= t_F(l) - \Delta t + w_F(l+1), \\ z_F(l+1) &= t_F(l+1) + v_F(l+1), \end{aligned} \quad (20)$$

where, $\Delta t = Mt_s$, again, for consecutive data records with no gap between them; $w_F(l)$ and $v_F(l)$ are assumed to be white, independent random variable with Gaussian distributions $p(w_F(l)) \sim N(0, Q_F(l))$ and $p(v_F(l)) \sim N(0, R_F(l))$, respectively. In this instance, we take $R_F(l) \propto \frac{dz_F(\phi(l), \phi_F)}{d\phi} \sigma_\phi^2(l)$ to be the covariance associated with each measurement $z_F(l)$, where $P_\phi(l)$ is a posteriori covariance for estimate of $\phi(l)$. Since the Eqs. (20) are linear we can again use the linear Kalman filter, in this case with time update equations:

$$\begin{aligned} \hat{t}_F^-(l+1) &= \hat{t}_F(l) - \Delta t, \\ P_F^-(l+1) &= P_F(l) + Q_F(l). \end{aligned} \quad (21)$$

The corresponding measurement update equations are:

$$\begin{aligned} K_F(l) &= \frac{P_F^-(l)}{(P_F^-(l) + R_F(l))}, \\ \hat{t}_F(l) &= \hat{t}_F^-(l) + K_F(l)(z_F(l) - \hat{t}_F^-(l)), \\ P_F(l) &= (1 - K_F(l))P_F^-(l). \end{aligned} \quad (22)$$

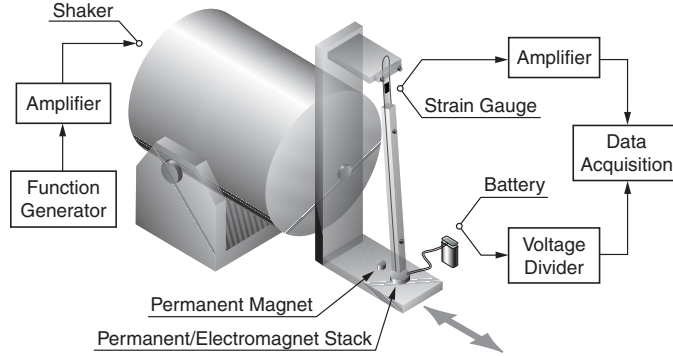


Figure 2. Schematic diagram of the experimental apparatus of the two-well electro-mechanical oscillator.

4. EXPERIMENTAL APPLICATION TO ELECTRO-MECHANICAL SYSTEM

The experimental system is a modification of a well known two-well magneto-mechanical oscillator.²⁴ It consists of a single degree-of-freedom cantilever beam with an assembly of permanent magnets and an electromagnet near its end, providing a two-well potential (see Fig. 2). The electromagnet is powered by a standard 9 volt battery. A strain gauge is attached to the beam below the fixture and just above the stiffeners added to the beam to ensure one mode vibrational response. The system is mounted on a shaker and is forced at about 6.3 Hz. The battery discharge process, which weakens the electromagnet strength, is viewed as a hidden damage process and the open circuit battery voltage is considered to be the damage variable. The forcing amplitude was set to obtain (possibly transient) chaotic response at least in the beginning of experiment. The experiment is started with a fresh 9 V battery and runs continuously for just under 7 hours. The stiffness in the electromagnet potential well decreases by a total of 6 percent due to the battery discharge. Strain gauge output is sampled at 100 Hz sampling frequency ($t_s = 0.01$ sec), digitized (12 bit A/D), and stored on a computer along with the independent voltage measurements taken from the electromagnets coil.

The system was chaotic throughout most of the experiment. However, numerous passages through windows of periodic behavior were also observed in the range of stiffnesses traversed by the system as the battery discharged. Delay time and embedding dimension for the reference data set were estimated to be $5t_s$ and 5, respectively. The first 2^{14} data points were used for the reference data set, and $N = 16$ nearest neighbors were used for the local linear model parameter estimation. The estimated average pointwise dimension of the reference data set was approximately $d_f = 2.82$.

After going through the embedding and modeling process we estimated the tracking function e_5 by calculating the short-time prediction error $E_5(l)$, which is a scalar in this case, of the reference model along with the distance r_l between the point $\mathbf{y}(l)$ to the farthest nearest neighbor points $\mathbf{y}^r(l)$ used in modeling. A sample distribution of $|E_5(l)|$ is highly asymmetrical and is shown on Fig. 3 (left). However, it can be approximated by a lognormal distribution. Indeed, if we take the natural log of the absolute value of the tracking function we get an approximately Gaussian distribution (Fig. 3 right). Thus, to estimate the tracking function $\hat{E}(l)$ we apply Kalman filter Eqs. (14)–(15) to $\ln(|E_5(l)|)$ and, then, transform it back using exponential function. The filter parameters we used are: $R(l) = 10^3 r_l^{d_f}$, $Q = 10^{-5}$, and $P^-(1) = 1$.

A mathematical model for the experimental system was developed elsewhere by the authors.¹⁶ It was shown that the following form of the battery voltage evolution law closely follows the experimental battery voltage data:

$$\dot{\phi} = -\epsilon(\phi - \psi) \left(1 + \gamma(\phi - \eta)^2 \right), \quad (23)$$

where $0 < \epsilon \ll 1$ is the rate constant, ψ represents residual voltage after battery discharge, η corresponds to the midpoint of the battery voltage operating range, and γ is some positive parameter.

To determine the parameters of the battery voltage model Eq. (23), we make use of independently measured electromagnet coil voltage data. We should note that the recorded battery voltage signal has a modulation induced by the inductive coupling of the electromagnet electrical circuit and the beam oscillations. We assume that the local

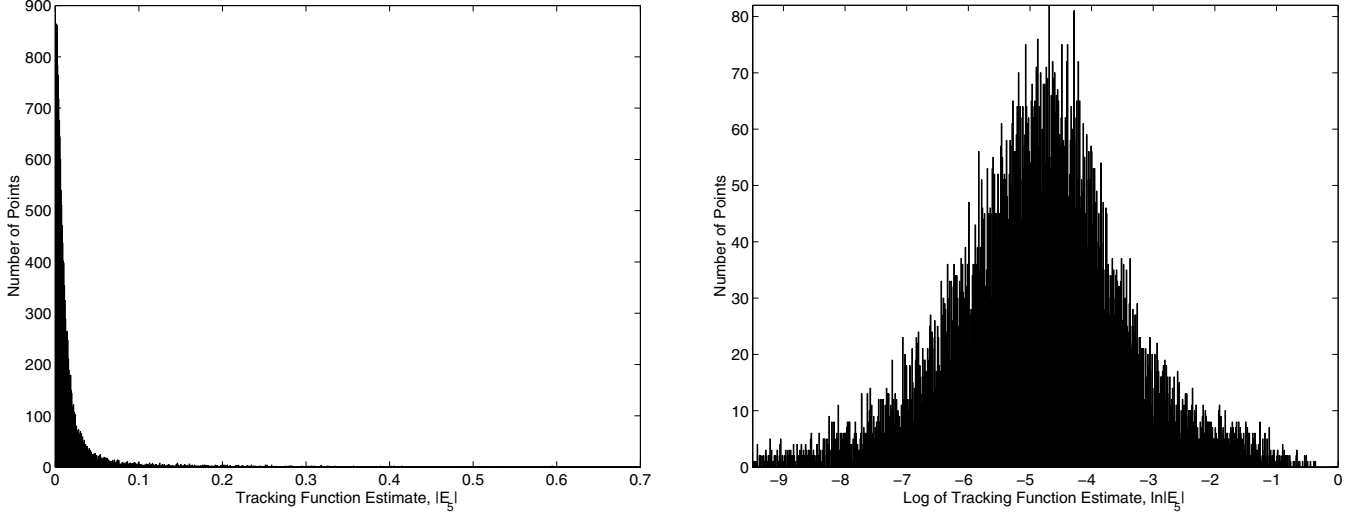


Figure 3. Probability distributions of $|E_5|$ (left) and $\ln|E_5|$ (right). 2^{14} points were used for histograms.

time average of the recorded voltage signal corresponds to the actual open circuit battery voltage. Fig. 4 (left plot) uses thick gray line to depict the local mean of the measured battery voltage. On the same plot we present results of nonlinear curve fitting of Eq. (23) to the recorded voltage data using dashed black line. The nonlinear curve fitting resulted in the following parameters $\epsilon = 0.0429$, $\psi = 0.0766$, $\gamma = 4.2050$, and $\eta = 5.9168$. These parameters would be used to formulate the discrete-time state transition equations for Eqs. (17). To determine the corresponding measurement equation we need to determine the linear fit parameters between the battery voltage data and the estimated tracking function. Using a linear polynomial fit for Eq. (5) we have: $e_5 = CV + c$, where $C = -0.0085$, $c = 0.073$, and V corresponds to local time mean of the measured voltage data. This resulted in the following form of Eqs. (17):

$$\begin{aligned} \phi(l+1) &= \phi(l) - \epsilon \Delta t (\phi(l) - \psi) \left(1 + \gamma (\phi(l) - \eta)^2\right) + w_\phi(l+1), \\ z_\phi(l+1) &= C\phi(l+1) + c + v_\phi(l+1), \end{aligned} \quad (24)$$

Consecutive data records of $M = 4,000$ points (with no overlapping sections or gaps between records) were used to calculate the mean $z_\phi(l) = e_5(l)$ and its variation $\sigma_{e_5}^2(l)$ using Eq. (16). Now, we use the nonlinear recursive filter for damage state estimation and Eqs. (24) to estimate the battery voltage, as shown in Fig. 4 (left plot) using a solid black line. In this case we used $R_\phi(l) = 10^2 \sigma_{e_5}^2(l)$, $Q_\phi = 10^{-2}$, and $P_\phi^-(1) = 1$.

By integrating Eq. (23) we can calculate the time elapsed from the initial to the current value of ϕ :

$$t(\phi) = -\frac{\ln \frac{\phi - \psi}{\sqrt{1 + \gamma(\phi - \eta)^2}} + (\eta - \psi)\sqrt{\gamma} \arctan[\sqrt{\gamma}(\phi - \eta)]}{\epsilon \left(1 + \gamma(\eta - \psi)^2\right)} + \frac{c}{\epsilon}, \quad (25)$$

We chose a failure value for the battery voltage of $\phi_F = 3.567 \text{ V}^\S$, and used the following expression as the measurement input in Eqs. (20):

$$z_F(l) = t(\phi_F) - t(\phi(l)), \quad (26)$$

where $\phi(l)$ is a current estimate of the battery voltage. Results of $z_F(l)$ calculation are shown in Fig. 4 (right plot) using a thin black line. To calculate corresponding variances we use the following equation:

$$R_F(l) \propto \left| \frac{dz_F(\phi(l))}{d\phi} \right| \sigma_\phi^2(l) = \frac{\sigma_\phi^2(l)}{\epsilon |\phi(l) - \psi| \left(1 + \gamma(\phi(l) - \eta)^2\right)} \quad (27)$$

[§]This value is arbitrary: it corresponds to the point where the first of the Eqs. (24) intersects the measured battery voltage. Changing it would not significantly change our results.

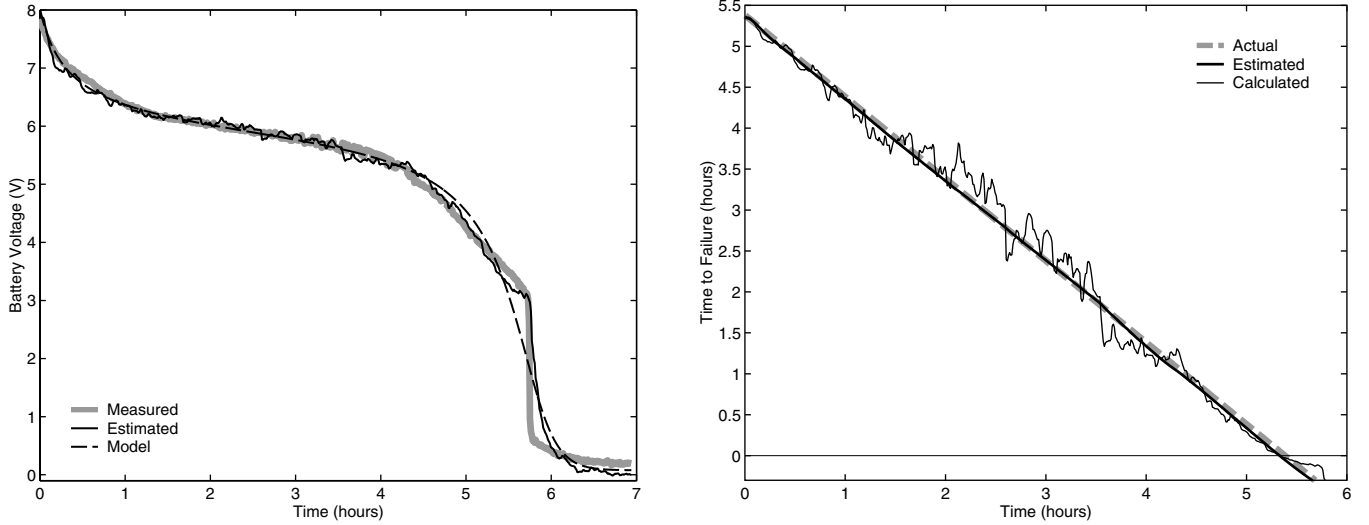


Figure 4. Damage state estimation and failure prognosis: (*left*) plot of local mean of measured battery voltage (heavy gray line), fitted nonlinear battery discharge model (dashed black line), and recursively estimated battery state (solid black line) vs. time; (*right*) time-to-failure predictions based on damage state estimates. In the time-to-failure predictions, the dashed heavy grey line indicates the true time to failure (known *a posteriori*), thin black line represents time-to-failure calculated using Eq. (26), and thick black line indicates the failure prognostic filter output.

Now, using Eqs. (20)–(22) we can estimate the remaining time-to-failure. Results of this calculation are shown using a thick black line in Fig. 4 (right plot). For this calculation, we have used $Q_F = 10^{-4}$, $P_F^-(1) = 1$. The standard deviation for each estimate was too small to resolve in the plot. For a fixed ϕ_F , the failure prediction algorithm was able to provide accurate estimates of the time-to-failure (known *a posteriori* and shown by the heavy gray dashed line in Fig. 4) throughout the whole experiment.

5. CONCLUSIONS

In this paper we have presented a new algorithm for estimating damage states and predicting failures in electro-mechanical systems. The algorithm is general purpose, since it is not tied to any specific damage physics. It assumes that the system possesses time scale separation, i.e., damage process occurs on a much slower time scale than the observable dynamics of the system. It is assumed implicitly that the fast directly observable subsystem is governed by some ordinary differential equation (Eq. 1a), however and explicit model need not be known. However, for failure prediction, a mathematical model for the damage evolution (Eq. 1b) is required.

We have described the major parts of our diagnostics/prognostics algorithm. In the first part, local linear reference models and short-time prediction error statistics are used to obtain a tracking metric. In the second part, we discussed how the tracking metric together with a damage evolution model can be used to estimate the current damage state. Finally, we explained how predictions of the remaining time-to-failure can be obtained.

The algorithm was applied to an electro-mechanical oscillator with a drifting mechanical potential energy, caused by an electromagnet powered by a discharging battery. The battery discharge was considered to be a “hidden” damage process. As the battery discharged to complete failure, the system’s stiffness in the potential energy well powered by the electromagnet declined by 6 percent. Throughout the experiment the system underwent many bifurcations causing repeated periodic/chaotic transitions. Nevertheless, the method smoothly tracked the battery voltage, and predicted the remaining useful life of the battery well in advance of actual failure, using only strain measurements taken from the fast subsystem.

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